

Name: \_\_\_\_\_  
A2CC: Remainder Theorem and Factor Theorems

Date: \_\_\_\_\_

Do now:

1. Let  $P(x) = 3x^5 + 5x^4 - 4x^3 + 7x + 3$ .
  - (a) Find the quotient and remainder when  $P(x)$  is divided by  $x + 2$ .
  - (b) Find  $P(-2)$ .

**Remainder Theorem:**

If the polynomial  $P(x)$  is divided by  $x - c$ , then the remainder is the value  $P(c)$ .

1. Let  $P(x) = x^3 - 2x^2 + 3x - 1$ . Find  $P(3)$  using 2 different methods.

**Factor Theorem:**

A polynomial  $P(x)$  has a factor of  $x - c$  if and only if  $P(c) = 0$ .

2. Show that  $x - 2$  is a factor of  $P(x) = x^3 - 3x^2 + 7x - 10$ .

3. (a) Use the factor theorem to show that  $x + 3$  is a factor of  $P(x) = x^3 - x^2 - 8x + 12$ .  
(b) Factor  $P(x)$  completely.

4. Let  $P(x) = x^3 - 7x + 6$ .  
(a) Show that  $P(1) = 0$ .  
(b) Factor  $P(x)$  completely.

5. Find a polynomial of degree 4 that has zeros  $-3, 0, 1$ , and  $5$ .

**THE REMAINDER THEOREM**  
**COMMON CORE ALGEBRA II**  
**EXERCISE SET A**

In the last lesson, we saw how two polynomials, when divided, resulted in another polynomial and a remainder. The remainder has a remarkable property in certain types of division. We will explore this relationship in the first exercise.

**Exercise #1:** Consider each of the following scenarios where we have  $\frac{p(x)}{x-a}$ . In each case, simplify the division using polynomial long division and then evaluate  $p(a)$ .

(a)  $\frac{x^2 - 8x + 18}{x - 2}$

$$p(x) = x^2 - 8x + 18 \Rightarrow p(2) =$$

(b)  $\frac{x^2 - 2x - 25}{x - 7}$

$$p(x) = x^2 - 2x - 25 \Rightarrow p(7) =$$

(c)  $\frac{2x^2 + 11x + 11}{x + 3}$

$$p(x) = 2x^2 + 11x + 11 \Rightarrow p(-3) =$$

(d)  $\frac{3x^2 + 7x - 20}{x + 4}$

$$p(x) = 3x^2 + 7x - 20 \Rightarrow p(-4) =$$



**Exercise #2:** If the ratio  $\frac{x^2 - 11x + 22}{x - 9}$  was placed in the form  $q(x) + \frac{r}{x - 9}$ , where  $q(x)$  is a linear function, then which of the following is the value of  $r$ ?

- (1) -3                                      (3) -9  
(2) 5                                        (4) 4

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In the past, the remainder theorem was used primarily to aid in evaluating polynomials. These days it is the primary justification for telling if a linear expression is a factor of a polynomial.

**Exercise #3:** By definition  $(x - a)$  is a factor of  $p(x)$  if  $\frac{p(x)}{x - a} = q(x)$ , where  $q(x)$  is another polynomial. What must be true of the remainder,  $p(a)$ , for  $(x - a)$  to be a factor of  $p(x)$ ? Explain.

**Exercise #4:** Determine if each of the following are factors of the listed polynomials by evaluating the polynomials.

- (a) Is  $x - 3$  a factor of  $p(x) = x^2 - 11x + 24$ ?                                      (b) Is  $x + 5$  a factor of  $p(x) = 2x^2 + 9x - 2$ ?  
(d) Is  $x + 1$  a factor of  $p(x) = x^3 - 7x^2 - 11x - 3$                                       (c) Is  $x - 5$  a factor of  $p(x) = x^3 - x^2 - 19x - 10$ ?

**Exercise #5:** For what value of  $k$  will  $x - 4$  be a factor of  $x^2 + kx - 52$ ? Show how you arrived at your answer.



**THE REMAINDER THEOREM**  
**COMMON CORE ALGEBRA II**  
**Exercise Set B**

**FLUENCY**

1. Which of the following is the remainder when the polynomial  $x^2 - 5x + 3$  is divided by  $(x - 8)$ ?
- (1) 107                      (3) 3  
(2) 27                      (4) 9
- \_\_\_\_\_
2. If the ratio  $\frac{2x^2 + 17x + 42}{x + 5}$  is placed in the form  $q(x) + \frac{r}{x + 5}$ , where  $q(x)$  is a polynomial, then which of the following is the correct value of  $r$ ?
- (1) -3                      (3) 18  
(2) 177                      (4) 7
- \_\_\_\_\_
3. When the polynomial  $p(x)$  was divided by the factor  $x - 7$  the result was  $x + \frac{11}{x - 7}$ . Which of the following is the value of  $p(7)$ ?
- (1) -8                      (3) 11  
(2) 7                      (4) It does not exist
- \_\_\_\_\_
4. Which of the following binomials is a factor of the quadratic  $4x^2 - 35x + 24$ ? Try to do this without factoring but by using the Remainder Theorem.
- (1)  $x + 6$                       (3)  $x - 8$   
(2)  $x - 4$                       (4)  $x + 2$
- \_\_\_\_\_
5. Which of the following linear expressions is a factor of the cubic polynomial  $x^3 + 9x^2 + 16x - 12$ ?
- (1)  $x + 6$                       (3)  $x - 3$   
(2)  $x - 1$                       (4)  $x + 2$
- \_\_\_\_\_



6. Consider the cubic polynomial  $p(x) = x^3 + x^2 - 46x + 80$ .

(a) Using polynomial long division, write the ratio of  $\frac{p(x)}{x-3}$  in **quotient-remainder form**, i.e. in the form

$q(x) + \frac{r}{x-3}$ . Evaluate  $p(3)$ . How does this help you check your quotient-remainder form?

(b) Evaluate  $p(5)$ . What does this tell you about the binomial  $x-5$ ?

(c) If  $q(x) = \frac{p(x)}{x-5}$ , then use polynomial long division to find an expression for the polynomial  $q(x)$ .

(d) Use your answer from (c) to **completely factor** the cubic polynomial  $p(x)$ . Besides  $x=5$ , what are its other zeroes?

7. For the cubic  $x^3 + 7x^2 + 13x + 3$  has only one rational zero,  $x = -3$ . Use polynomial long division to show that the remainder is zero when dividing the cubic by  $x+3$ . Then use the quadratic formula to find the other two (irrational) zeroes.

